UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the May/June 2007 question paper

9709 MATHEMATICS

9709/03 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the May/June 2007 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only - often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures - this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1 *EITHER:* Obtain correct unsimplified version of the x or x^2 term in the expansion of $(2+3x)^{-2}$ or $(1+\frac{3}{2}x)^{-2}$ M1

State correct first term $\frac{1}{4}$ B1

Obtain the next two terms $-\frac{3}{4}x+\frac{27}{16}x^2$ A1 + A1

[The M mark is not earned by versions with symbolic binomial coefficients such as $\binom{-2}{1}$.]

[The M mark is earned if division of 1 by the expansion of $(2+3x)^2$, with a correct unsimplified x or x^2 term, reaches a partial quotient of $a + bx$.]

[Accept exact decimal equivalents of fractions.]

[SR: Answer given as $\frac{1}{4}(1-3x+\frac{27}{4}x^2)$ can earn B1M1A1 (if $\frac{1}{4}$ seen but then omitted, give M1A1).]

[SR: Solutions involving $k(1+\frac{3}{2}x)^{-2}$, where $k = 2$, 4 or $\frac{1}{2}$, can earn M1and A1√ for correctly simplifying both the terms in x and x^2.]

OR: Differentiate expression and evaluate f(0) and f′(0), where $f'(x) = k(2+3x)^{-3}$ M1

State correct first term $\frac{1}{4}$ B1

Obtain the next two terms $-\frac{3}{4}x+\frac{27}{16}x^2$ A1 + A1 **4**

2 **(i)** Substitute $x = -2$ and equate to zero, or divide by $x + 2$ and equate constant remainder to zero, or use a factor $Ax^2 + Bx + C$ and reach an equation in a M1

Obtain answer $a = 4$ A1 **2**

(ii) Attempt to find quadratic factor by division or inspection M1

State or exhibit quadratic factor $x^2 - 2x + 2$ A1 **2**

[The M1 is earned if division reaches a partial quotient $x^2 + kx$, or if inspection has an unknown factor $x^2 + bx + c$ and an equation in b and/or c, or if inspection without working states two coefficients with the correct moduli.]

3 Use product rule M1

Obtain derivative in any correct form A1

Form equation of tangent at $x = \frac{1}{4}\pi$ correctly M1

Simplify answer to $y = x$, or $y - x = 0$ A1 **4**

[SR: The misread $y = x\sin x$ can only earn M1M1.]

4 State or imply at any stage that $3^{-x} = \frac{1}{3^x}$, or that $3^{-x} = \frac{1}{u}$ where $u = 3^x$ B1

Convert given equation into the 3-term quadratic in u (or 3^x): $u^2 - 2u - 1 = 0$ B1

Solve a 3-term quadratic, obtaining one or two roots M1

Obtain root $\frac{2+\sqrt{8}}{2}$, or a simpler equivalent, or decimal value in [2.40, 2.42] A1

Use a correct method for finding the value of x from a positive root M1

Obtain $x = 0.802$ only A1 **6**

5 **(i)** State answer $R = 2$
Use trig formula to find α
Obtain $\alpha = \frac{1}{3}\pi$, or 60° A1

[For the M1 condone a sign error in the expansion of $\cos(\theta - \alpha)$, but the subsequent trigonometric work must be correct.]

[SR: The answer $\alpha = \tan^{-1}(\sqrt{3})$ earns M1 only.]

(ii) State that the integrand is of the form $a\sec^2(\theta - \alpha)$ M1

State correct indefinite integral $\frac{1}{4}\tan(\theta - \frac{1}{3}\pi)$ A1√

Use limits correctly in an integral of the form $a\tan(\theta - \alpha)$ M1

Obtain given answer correctly following full and exact working A1 **4**

[The f.t. is on R and α.]

6 **(i)** Using the formulae $\frac{1}{2}r^2\alpha$ and $\frac{1}{2}r^2\sin\alpha$, or equivalent, form an equation M1

Obtain given equation correctly A1 **2**

[Allow the use of OA and/or OB for r.]

(ii) Consider sign of $x - 2\sin x$ at $x = \frac{1}{2}\pi$ and $x = \frac{2}{3}\pi$, or equivalent M1

Complete the argument correctly with appropriate calculations A1 **2**

(iii) State or imply the equation $x = \frac{1}{3}(x + 4\sin x)$ B1

Rearrange this as $x = 2\sin x$, or work *vice versa* B1 **2**

(iv) Use the iterative formula correctly at least once M1

Obtain final answer 1.90 A1

Show sufficient iterations to 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (1.895, 1.905) A1 **3**

[The final answer 1.9 scores A0.]

7 **(i)** State or imply $du = \dfrac{1}{2\sqrt{x}}dx$, or $2u\,du = dx$, or $\dfrac{du}{dx} = \dfrac{1}{2\sqrt{x}}$, or equivalent B1

Substitute for x and dx throughout the integral M1

Obtain the given form of indefinite integral correctly with no errors seen A1 **3**

(ii) Attempting to express the integrand as $\dfrac{A}{u} + \dfrac{B}{4-u}$, use a correct method to find either A or B M1*

Obtain $A = \frac{1}{2}$ and $B = \frac{1}{2}$ A1

Integrate and obtain $\frac{1}{2}\ln u - \frac{1}{2}\ln(4-u)$, or equivalent A1√ + A1√

Use limits $u = 1$ and $u = 2$ correctly, or equivalent, in an integral of the form $c\ln u + d\ln(4-u)$ M1(dep*)

Obtain given answer correctly following full and exact working A1 **6**

8	(i) *EITHER*:	Carry out multiplication of numerator and denominator by $-1-i$, or solve for x or y		
		Obtain $u = -1 - i$, or any equivalent of the form $(a + ib)/c$		
		State modulus of u is $\sqrt{2}$ or 1.41	A1	
		State argument of u is $-\frac{3}{4}\pi$ (−2.36) or −135°, or $\frac{5}{4}\pi$ (3.93) or 225°	A1	
	OR:	Divide the modulus of the numerator by that of the denominator	M1	
		State modulus of u is $\sqrt{2}$ or 1.41	A1	
		Subtract the argument of the denominator from that of the numerator, or equivalent	M1	
		State argument of u is $-\frac{3}{4}\pi$ (−2.36) or −135°, or $\frac{5}{4}\pi$ (3.93) or 225°	A1	
		Carry out method for finding the modulus or the argument of u^2	M1	
		State modulus of u is 2 and argument of u^2 is $\frac{1}{2}\pi$ (1.57) or 90°	A1	6
	(ii)	Show u and u^2 in relatively correct positions	B1√	
		Show a circle with centre at the origin and radius 2	B1	
		Show the line which is the perpendicular bisector of the line joining u and u^2	B1√	
		Shade the correct region, having obtained u and u^2 correctly	B1	4
9	(i) *EITHER*:	Obtain a vector parallel to the plane, e.g. $\overrightarrow{AB} = -\mathbf{i} + 2\mathbf{j}$	B1	
		Use scalar product of perpendicular vectors to obtain an equation in a, b, c, e.g. $-a + 2b = 0$, or $-a + b + 2c = 0$, or $-b + 2c = 0$	M1	
		Obtain two correct equations in a, b, c	A1	
		Solve to obtain ratio $a : b : c$, or equivalent	M1	
		Obtain $a : b : c = 4 : 2 : 1$, or equivalent	A1	
		Obtain equation $4x + 2y + z = 8$, or equivalent	A1	
	*OR*1:	Substitute for A and B and obtain $2a = d$ and $a + 2b = d$	B1	
		Substitute for C to obtain a third equation and eliminate one unknown (a, b, or d) entirely	M1	
		Obtain two correct equations in three unknowns, e.g. a, b, c	A1	
		Solve to obtain their ratio, e.g. $a : b : c$, or equivalent	M1	
		Obtain $a : b : c = 4 : 2 : 1$, or $a : c : d = 4 : 1 : 8$, or $b : c : d = 2 : 1 : 8$, or equivalent	A1	
		Obtain equation $4x + 2y + z = 8$, or equivalent	A1	
	*OR*2:	Substitute for A and B and obtain $2a = d$ and $a + 2b = d$	B1	
		Solve to obtain ratio $a : b : d$, or equivalent	M2	
		Obtain $a : b : d = 2 : 1 : 4$, or equivalent	A1	
		Substitute for C to find c	M1	
		Obtain equation $4x + 2y + z = 8$, or equivalent	A1	
	*OR*3:	Obtain a vector parallel to the plane, e.g. $\overrightarrow{BC} = -\mathbf{j} + 2\mathbf{k}$	B1	
		Obtain a second such vector and calculate their vector product, e.g. $(-\mathbf{i} + 2\mathbf{j}) \times (-\mathbf{j} + 2\mathbf{k})$	M1	
		Obtain two correct components of the product	A1	
		Obtain correct answer, e.g. $4\mathbf{i} + 2\mathbf{j} + \mathbf{k}$	A1	
		Substitute in $4x + 2y + z = d$ to find d	M1	
		Obtain equation $4x + 2y + z = 8$, or equivalent	A1	
	*OR*4:	Obtain a vector parallel to the plane, e.g. $\overrightarrow{AC} = -\mathbf{i} + \mathbf{j} + 2\mathbf{k}$	B1	
		Obtain a second such vector and form correctly a 2-parameter equation for the plane	M1	
		Obtain a correct equation, e.g. $\mathbf{r} = 2\mathbf{i} + \lambda(-\mathbf{i} + 2\mathbf{j}) + \mu(-\mathbf{i} + \mathbf{j} + 2\mathbf{k})$	A1	
		State three equations in x, y, z, λ, μ	A1	
		Eliminate λ and μ	M1	
		Obtain equation $4x + 2y + z = 8$, or equivalent	A1	6
	(ii)	State or imply a normal vector for plane OAB is $\mathbf{k}$, or equivalent	B1	
		Carry out correct process for evaluating a scalar product of two relevant vectors, e.g. $(4\mathbf{i} + 2\mathbf{j} + \mathbf{k}).(\mathbf{k})$	M1	
		Using the correct process for calculating the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result	M1	
		Obtain answer 77.4° or 1.35 radians	A1	4

10 (i) State $\frac{dh}{dt} = k(9-h)^{\frac{1}{3}}$

Show that $k = 0.1$ B1

(ii) Separate variables correctly and attempt integration of at least one side M1

Obtain terms $-\frac{3}{2}(9-h)^{\frac{2}{3}}$ and $0.1t$, or equivalent A1 + A1

Evaluate a constant, or use limits $t = 0$, $h = 1$ with a solution containing terms of the form $a(9-h)^p$ and bt, where $p > 0$ M1*

Obtain solution in any form, e.g. $-\frac{3}{2}(9-h)^{\frac{2}{3}} = 0.1t - 6$ A1

Rearrange and make h the subject M1(dep*)

Obtain answer $h = 9-(4-\frac{1}{15}t)^{\frac{3}{2}}$, or equivalent A1 **7**

(iii) State that the maximum height is $h = 9$ B1

State that the time taken is 60 years B1 **2**

(iv) Substitute $h = 9/2$ and obtain $t = 19.1$ (accept 19, 19.0 and 19.2) B1 **1**